Zeeman Effect

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1 Abstract

In this experiment, we will observe the Zeeman effect which is defined as splitting of transition lines in the presence of magnetic field. We will observe the Zeeman effect for mercury lamp by the application of a magnetic field and exciting transitions by placing a high voltage across the sample. We will analyze our results by trying to measure the extent of splitting.

2 Introduction

Most of the atoms have several electronic configurations at same energy such that transitions between these electronic configurations correspond to single spectral line. Application of magnetic field breaks this degeneracy. So, when a magnetic field is applied to an atom that is emitting photons, the field distorts the electron orbitals, causing the frequencies of light emitted to be split into distinct spectral lines with different energies. This is known as Zeeman effect [1]. When the splitting occurs into two or three lines, it is called normal Zeeman effect and can be explained quantitatively by classical theory. The splitting of a spectral line into more than three components in ordinary weak magnetic fields is called anomalous Zeeman effect.

It was Pieter Zeeman who discovered this effect in 1896 and won nobel prize. The normal Zeeman effect was successfully explained by H. A. Lorentz using the laws of classical physics. The anomalous Zeeman effect could not be explained using classical physics [2]. It could only be explained using quantum theory and taking into account the electron's intrinsic spin. According to the quantum theory, all spectral lines arise from transitions of electrons between different allowed energy levels within the atom, the frequency of the spectral line are proportional to the energy difference between the initial and final levels. Because of its intrinsic spin, the electron has a magnetic field associated with it. When an external magnetic field is applied, the electron's magnetic field may assume only certain alignments. Slight differences in energy are associated with these different orientations, so that what was once a single energy level becomes three or more [3].

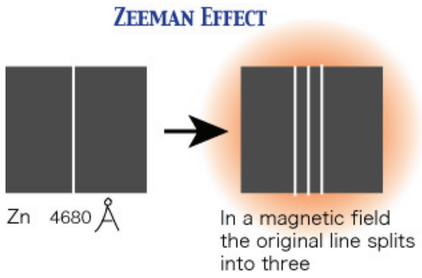


Figure 1: Zeeman effect for mercury lamp

3 Theoretical Background

3.1 Zeeman Effect

To understand the idea of Zeeman effect, consider a magnetic dipole placed inside an external magnetic field. Now, the total Hamiltonian of the system becomes

$$H = H_0 + H_\mu \tag{1}$$

where H_0 is unperturbed Hamiltonian and H_μ is defined as:

$$H_\mu = -\mu.B \tag{2}$$

where μ is inherent magnetic dipole moment of the atom [4].

We know from the structure of an atom that there are two distinct electronic magnetic dipole moments. The first moment μ_S is due to the spin angular momentum and the other μ_L is due to the orbital motion of the electron around the nucleus. If the quantum operator for spin angular momentum is $\overrightarrow{S}$ and $\overrightarrow{L}$ for angular momentum, then we have the equations:

$$\overrightarrow{\mu_S} = \gamma \overrightarrow{S} = -\frac{g_e e}{2m_e}\overrightarrow{S} = -\frac{e}{m_e}\overrightarrow{S} \tag{3}$$

Here γ is gyromagnetic ratio and g_e is a Lande g$-$factor of electron which has a value of 2 in case of spin angular momentum. Similarly,

$$\overrightarrow{\mu_L} = \gamma \overrightarrow{L} = -\frac{g_e e}{2m_e}\overrightarrow{S} = -\frac{e}{2m_e}\overrightarrow{L} \tag{4}$$

For angular momentum of electron, Lande g−factor has a value of 1.

Now total magnetic moment becomes:

$$\overrightarrow{\mu} = \overrightarrow{\mu_S} + \overrightarrow{\mu_L} = -\frac{e}{2m}(\overrightarrow{L} + 2\overrightarrow{S}) \tag{5}$$

So, from equation (1), the perturbed Hamiltonian is given by:

$$H_{Zeeman} = \frac{e}{2m}(\overrightarrow{L} + 2\overrightarrow{S})\overrightarrow{B_{ext}} \tag{6}$$

Since, total momentum $\overrightarrow{J}$ is a sum of spin angular momentum $\overrightarrow{S}$ and orbital angular momentum $\overrightarrow{L}$ so we can write the above equation as

$$H_{Zeeman} = \frac{e}{2m}(\overrightarrow{J} + \overrightarrow{S})\overrightarrow{B_{ext}} \tag{7}$$

To evaluate $< \overrightarrow{J}.\overrightarrow{B} >$ is easy. Evaluating $< \overrightarrow{S}.\overrightarrow{L} >$ needs a change of basis.

$$\overrightarrow{J}^2 = (\overrightarrow{L} + \overrightarrow{S})^2 \tag{8}$$

expanding and rearranging it

$$\overrightarrow{S}.\overrightarrow{J} = \frac{1}{2}(\overrightarrow{J}^2 - \overrightarrow{L}^2 + \overrightarrow{S}^2) \tag{9}$$

Which gives us the result

$$E_{Zeeman} = \mu_B B m_j g_j \tag{10}$$

where Bohr magneton $\mu_B = \frac{e\hbar}{2m}$
and $g_j = 1 + \frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)}$ known as Lande g-factor as mentioned earlier [5].

Based on the different values of m_j different values of energies are obtained. Since m_j runs from $-j$ to j, magnetic field splits each energy level into $2j + 1$.[3] That is the simple explanation of Zeeman effect.

Lets see how Zeeman effect happens for mercury.

The green line of mercury at $546nm$ arises from a transition between S_1^3 to P_2^3 Allowed transitions are given by the selection rule $\Delta m_j = 0, \pm 1$ This gives us 9 lines of transition as shown in figure. We can use a polarizer to see only 3 lines with $\Delta m_j = 0$

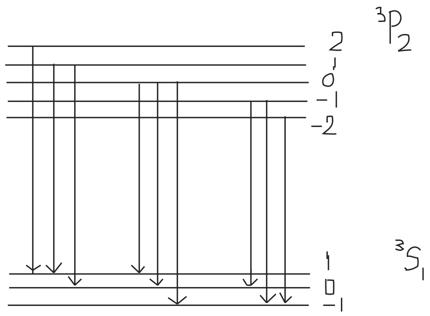


Figure 2: Transition lines for 546nm line of mercury lamp

3.2 Fabry Perot Interferometer

A Fabry-Perot interferometer is essentially two parallel highly reflective glass plates. The low transmission rate of light through the plates allows the light to reflect between the plates multiple times before emerging. The interference occurs and only one particular wavelength emerge axially centered with others forming a circular pattern around the optical axis.

If the light passes through the first plate at an angle θ and the plates separation is d then condition for constructive interference is

$$2d\cos\theta = m\lambda \tag{11}$$

for some integer m. The geometrical retracing of fabry-Perot interferometer is shown in figure (3)

The co-efficient of finesse for Fabry-Perot interferometer is given by:

$$F = \frac{4R}{(1-R)^2} \tag{12}$$

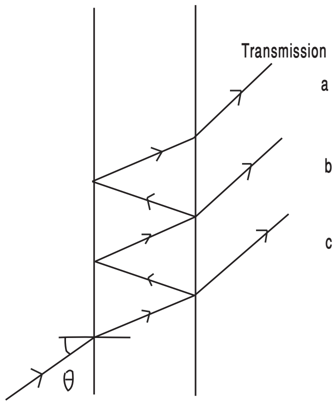
where R is the reflectance of both mirrors.

The wavelength separation between adjacent transmission peaks is called the free spectral range (FSR) of the etalon and is related to the full-width half-maximum of any one transmission band by a quantity known as the finesse.

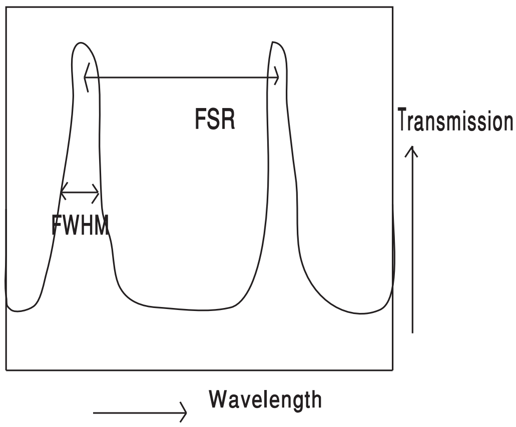
$$\overline{F} = \frac{FSR}{FWHM} \tag{13}$$

where $FWHM$ is full width of the peak at half maximum. Finesse is also approximated by

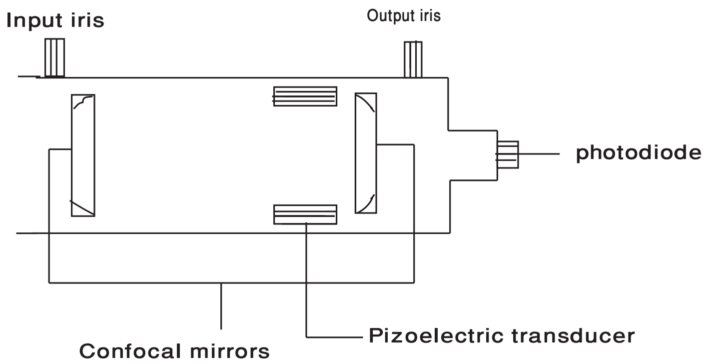
$$\overline{F} = \pi\frac{\sqrt{R}}{1-R} \tag{14}$$



a. A Fabry-Perot etalon



b. Transmission as a function of wavelength



c. Schematic of Fabry-Perot etalon

Figure 3: Fabry-Perot etalon

Etalons with high finesse show sharper transmission peaks with lower minimum transmission coefficients.

One of the plates is mounted on three micrometers, which we can adjust according to our choosing. The other plate is mounted on piezoelectric crystals, whose thicknesses change with the applied voltage. These crystals are controlled by a ramp generator, which sends slow sawtooth voltage ramps to those crystals, adjusting the plate separation and thus the wavelength which constructively interferes and is axially centered. By applying a large enough voltage ramp, we can sweep over a whole group of transitions. These optics allow for the naked-eye observation of Zeeman splitting.

4 Experimental procedure

Following steps were followed during this experiment.

4.1 Finding the Finesse of Fabry Perot Interferometer

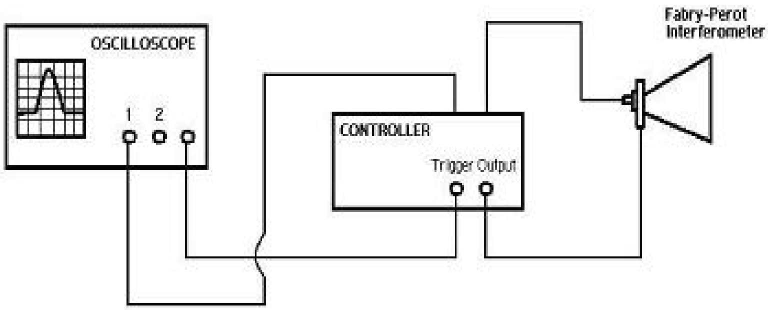
First step is to find Free spectral range Finesse of interferometer

1. An optical rail is fixed on the optical board to let the Fabry Perot interferometer move freely in the vertical palne. This is necessary for doing perfect alignment.
2. SA200 Fabry Perot interferometer was mounted on the mount such that it can be moved up and down and this assembly was fixed on the optical rail. He Ne laser is used for alignment. It was made sure that laser light is parallel to the optical board. Then, laser light was made to fall at exact center of the opening iris of the interferometer. The rest of the alignment was made from tip-tilt arrangement of the interferometer carefully.
3. Once alignment is done, connections are made as shown in the figure (4). As the controller scans the light coming from laser, the spectrum can be seen on the oscilloscope. Results are recorded for different sweep expansions..

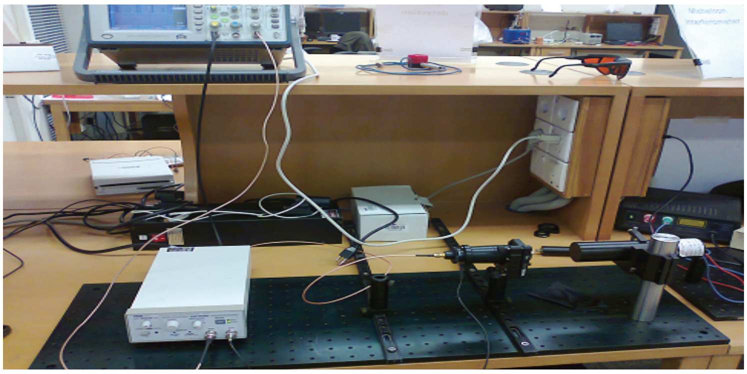
4.2 Results and Calculations

Figure(5) shows the results. We have different spectrum and positions of peaks for different sweep expansions.

From the figures, we do the following calculations:
The given FSR of the interferometer is $1.5GHz$. We look for the peaks in figures whose difference corresponds to FSR of the interferometer and by comparing both distances we measure FWHM and find finnesse.
For example, in figure (5c), counting from right to left, difference between peaks 1 and 5 corresponds to FSR. And that distance is $27ms$ on the oscilloscope. Comparing with this value FWHM is found out to be 1.43×10^7.



a. Schematic diagram for finding finesse of Fabry Perot Interferometer



b. Experimental Arrangement

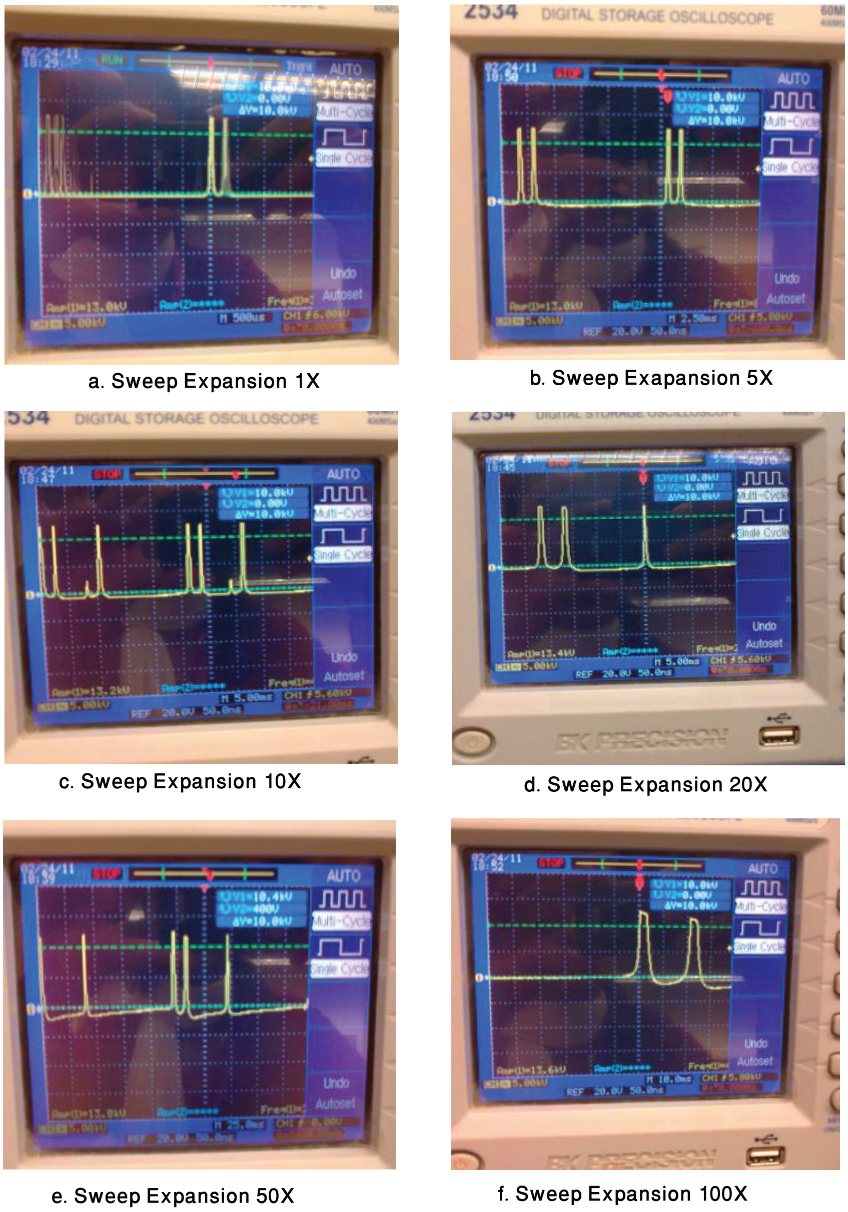
Figure 4: Experimnetal setup for Fabry-Perot interferometer

Thus, finnesse is: $\overline{F} = \frac{FSR}{FWHM} = \frac{1.5\times10^{9}}{1.43\times10^{7}} = 104$
Which is an acceptable value. Repeated calculations for Finesse give a value of 100 to 120.
Now, using equation (14), we can find the value of reflectivity for this value of finesse and that comes out to be
$\underline{F} = 104 = \frac{\pi\sqrt{R}}{1-R}$
$R = 0.99$

4.3 Spectrum of Mercury lamp

The spectrum of mercury lamp was observed following the steps written below:

1. Laser light was directional and intense but light from the mercury lamp is spreading around, it is hard to concentrate it on one point. Thus, oscilloscope cannot detect any intensity reaching to it. A spectrometer would be used to observe the spectrum of mercury lamp.
2. A BLUEWAVE SpectraWiz Spectrometer is used for this purpose. Software is installed and spectrum at different points of the lamp are recorded using the optical fiber of the spectrometer.



a. Sweep Expansion 1X
b. Sweep Exapansion 5X
c. Sweep Expansion 10X
d. Sweep Expansion 20X
e. Sweep Expansion 50X
f. Sweep Expansion 100X

Figure 5: Oscilloscope's output for different sweep expansion

3. As it is obvious from spectrum that at around 550nm, a quite intense peak is obtained. So, a green filter is used to get light of one wavelength so that it could interfere and give us an interference pattern. Result is recorded after putting a 500nm band pass filter and as we can see only one wavelength is allowed, others are blocked.

4.4 Calibration of magnetic field

Next step is to calibrate magnetic field.

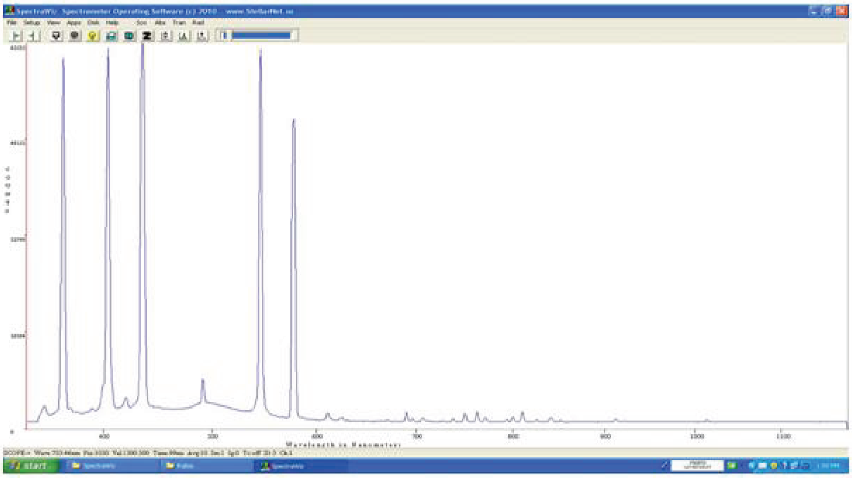


Figure 1: Spectrum of mercury lamp without filter

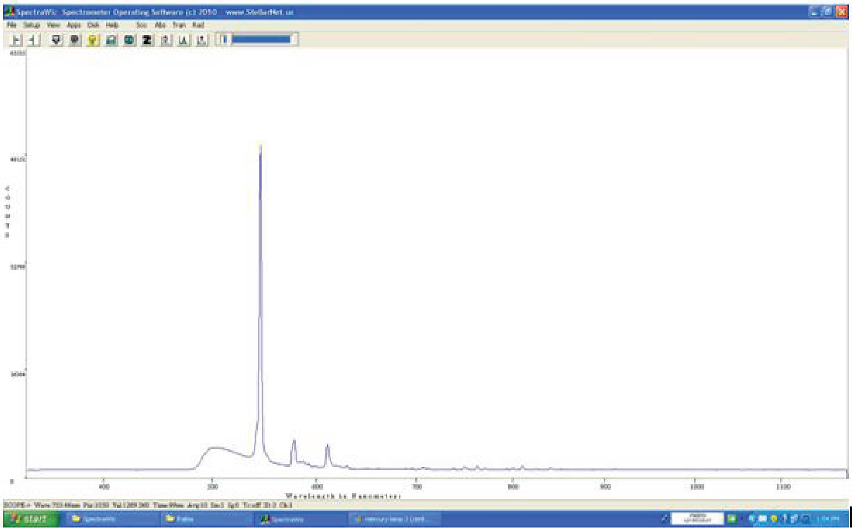


Figure 2: Spectrum of mercury lamp with green filter

Figure 6: Spectrum of mercury lamp

The space between the poles of the electromagnet is adjusted to fit mercury lamp in the center. Now the electromagnet has the following specifications.

$$\text{Pole gap (Cylindrical ends)} = 5cm$$
$$\text{Pole gap (Tapered ends)} = 3cm$$
$$\text{Standard pole face (cylindrical end)} = 4\text{cm}$$
$$\text{pole diameter} = 4.5cm$$

Specifications of the coil:

$$\text{Resistance at room temperature} = 7.3\Omega$$
$$\text{Maximum Resistance} = 8.8\Omega$$

Maximum power with air as cooling medium:

$$\text{Current} = 3.5A, \text{ Voltage} = 31V, \text{ Power} = 0.11kV$$

Maximum power with chilled water as cooling medium:

$$\text{Current} = 5A, \text{ Voltage} = 44V, \text{ Power} = 0.22kV$$

1. To measure magnetic field, Gaussmeter is used. I will be using transverse probe. It measures magnetic field perpendicular to the probe axis based on the principle of hall effect. The transverse probe of the gaussmeter is clamped such that the tip of the probe is at the exact center of the coils, where we have to put our mercury lamp.
2. Any ferromagnetic substance or magnetically sensitive item is removed from the optical board.
3. Chiller of the electromagnet is switched on. Water flow is checked. The pressure and temperature have the following values:

$$\text{Pressure} = 0.3Barr, \text{ Temperature} = 20^0C$$

4. Function generator is switched on. With the increase in the values of voltage and current, magnetic field increases which is measured by gaussmeter as mentioned earlier. Data is recorded in table 1. There is a linear relationship between current and magnetic field as plotted in matlab in figure 7..

Sr.no.	Voltage (Volts)	Current (Amps)	Reading 1	Reading 2	Mean B (kGauss)
1	0	0	0.0	0.028	0.014
2	4.0	0.50	0.510	0.536	0.523
3	8.0	1.00	1.008	1.039	1.023
4	12.0	1.50	1.508	1.54	1.522
6	16.0	2.00	2.01	2.04	2.025
7	19.9	2.50	2.51	2.54	2.525
8	23.9	3.00	3.01	3.04	3.025
9	27.8	3.50	3.50	3.53	3.525
10	32.0	4.00	3.98	4.01	3.995
11	36.0	4.50	4.45	4.47	4.46
12	40.1	5.00	4.85	4.85	4.85

Table 1: Relationship between Current and Magnetic Field

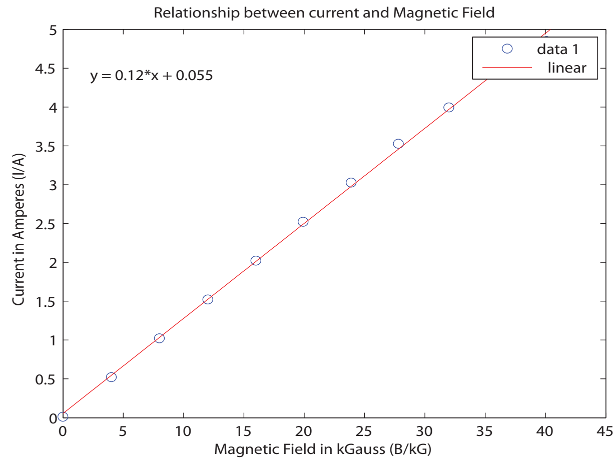


Figure 7: Increase in magnetic field with increase in current

4.5 Using mercury lamp to observe Zeeman effect

Finally, next step is to observe Zeeman effect. The experimental procedure is written down:

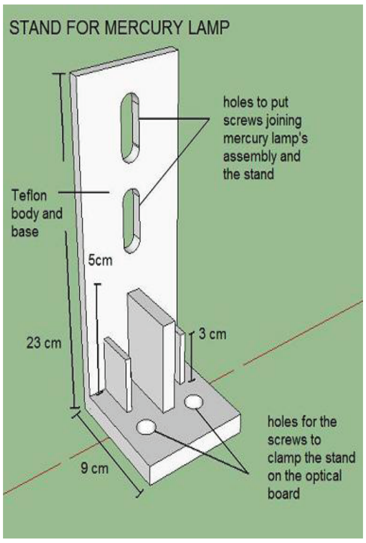
1. Initially, a simple clamp and iron stand was used to hold mercury lamp vertically in its position but then it was to put between magnetic field where no magnetic material can be used so a Teflon stand is designed and made for the lamp as shown in figure (8). The stand is fixed on the optical board with the help of screws.

2. A CCD camera is used to view the results. Software is installed and camera is connected with the computer. That appears to be pretty useful as we can view on the screen clearly whatever is happening. CCD camera has different available eyepieces. I would be using a lens of 25mm focal length.

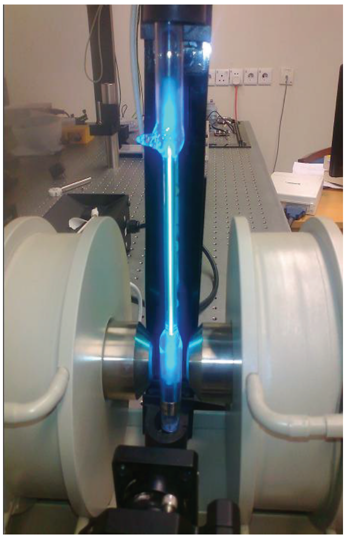
3. Another lens of 25mm focal length is used to focus light from mercury lamp onto the Fabry Perot interferometer. Laser beam and an external iris were used to make sure that everything is at exact same height and every component in the assembly is aligned. Alignment is done by adjusting the height of beam making it fall at the exact center of the lens, interferometer and camera. Further alignment is done by tip tilt arrangement of the interferometer. The circular fringes of the He Ne laser are visible on the computer screen.

4. Once alignment is done perfectly, laser assembly is replaced by mercury lamp and electromagnet with great care.

5. A telescope assembly is used to focus the image. Different combinations are tried making sure that the distance between light source and lens is same as the focal length of the lens. CCD with varying focal length and intensity is used. Other parts of lamp are covered with a cardboard such that no light falls on the lens of the CCD. Everything is done in dark room with no outside light.

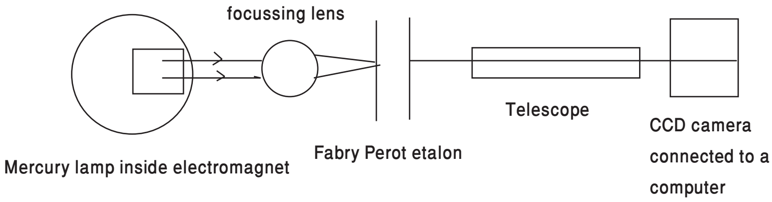


a. Mercury lamp stand made in google sketch-up

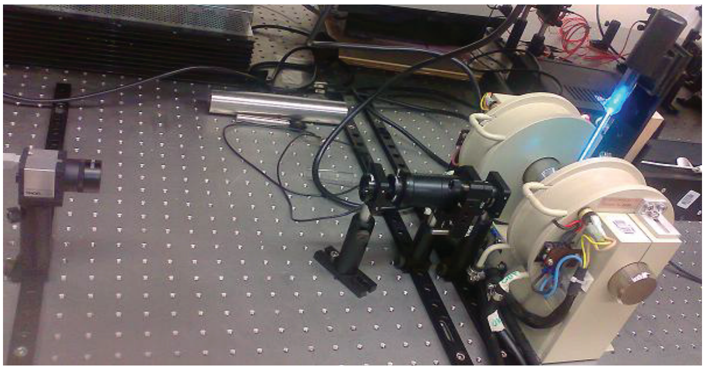


b. Mercury lamp clamped between the poles of the magnet by the stand

Figure 8: Stand of Mercury lamp



a. Schematic of experimental setup



b. Experimental Arrangement

Figure 9: Experimental Setup for Zeeman effect

6. With great difficulty, spending lot of time on alignment and re-alignment, trying different combinations of lenses to get the image, finally an interference

pattern is observed. Three rings are visible in the interference pattern.

7. With the increase in magnetic field, the behaviour of the fringes is observed written in results with detail.

4.6 Results and Conclusion

When magnetic field is increased, as discussed above the lines of the spectrum split into further lines as well as the overall intensity of the spectrum increases. If a polarizer is used, it selects specific lines among the split lines and that makes lines countable, but for now no polarizer has been used. All the lines merge up and the splitting cannot be measured. Results are recorded in figure (10).

5 Further Discussion

- A very interesting fact observed during the experiment is that increasing the magnetic field increases the intensity of light components as seen by the spectrometer on the computer screen. Even in the fringe pattern seen, intensity increases and spectrum appears brighter. The increase in intensity does not follow any particular relationship but there is a noticeable change in the spectrum. This can be observed by fixing the spectrometer infront of the mercury lamp and gradually increasing magnetic field. I chose two lines from the spectrum with wavelengths $575nm$ and $820nm$ and plotted their intensity versus magnetic field. Result shown in figure (11). Green dots show increase in intensity of line at $570nm$ and red dots show the increase in intensity of the spectrum line at $820nm$ which had almost zero intensity in the absence of magnetic field.

  The intensity goes high and the results remain almost same regardless of the direction of magnetic field. Somehow the frequency of the magnetic field interacts in such a way that more transition levels are produced which in turn increase the excitation and consequently number of emitted electrons.

- No filter was of any use in my experiment. A long pass filter of $400nm$ was not blocking anything and that of $500nm$ did not let any light pass through it. When the spectrum of the light reaching the CCD camera was observed through spectrometer, this seemed to be in complete agreement. There are no lines above $450nm$. So, all other spectrum lines are already blocked and do not reach the detector. They might have undergone multiple reflections inside the Fabry-Perot etalon and are completely cancelled out. This is strange though. The spectrum is shown in figure(12).

- Seeing the fringes and interference pattern was really difficult. Alignment of the setup is a bit tricky and demands extreme attention and care. The experiment was done in dark and the lamp was covered except for a small part to prevent direct light from falling onto camera as well as to minimize interference effects from different parts. It took great time to see the pattern

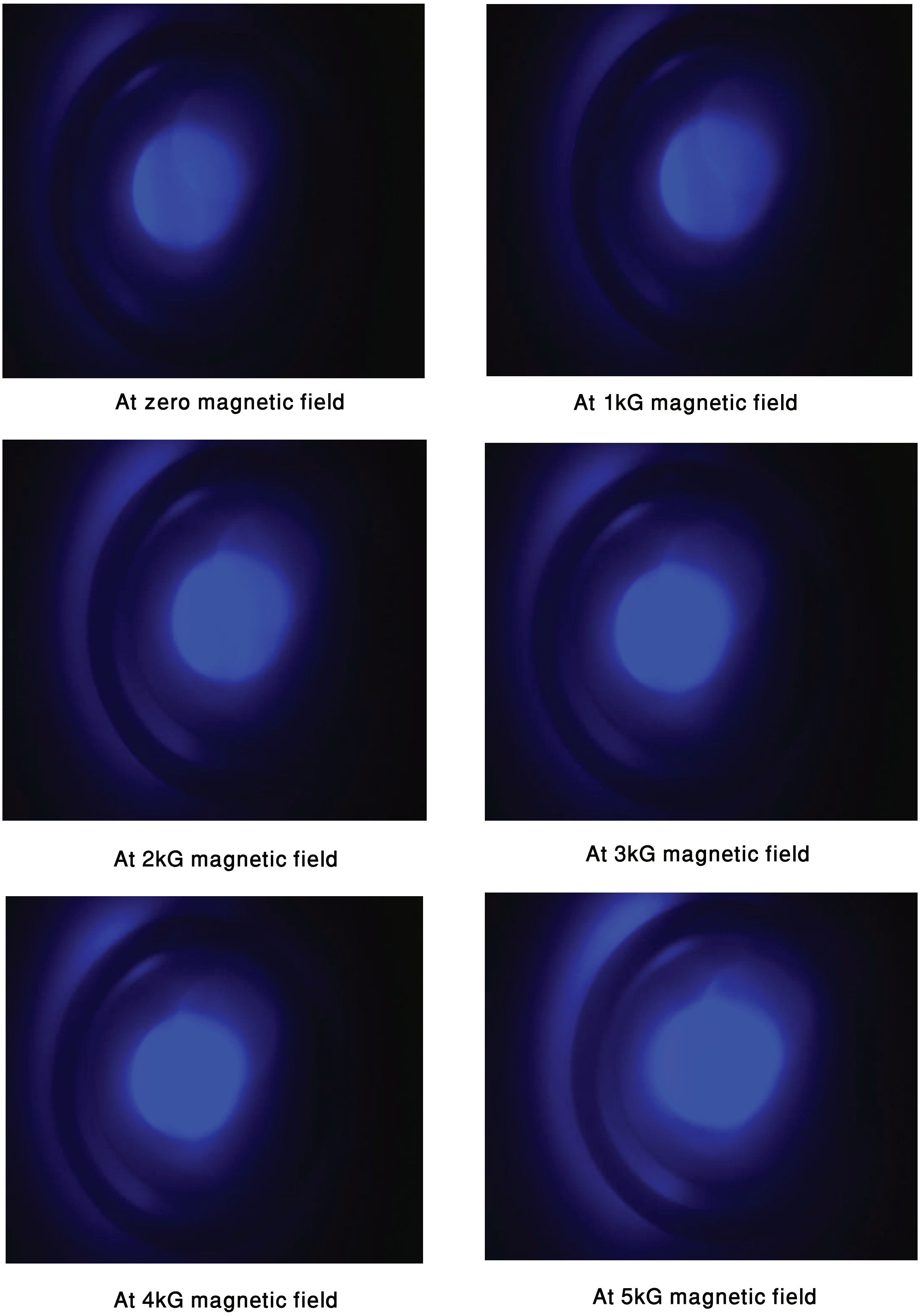


Figure 10: Zeeman effect

which is still not very good as expected. Also, due to the limitations of

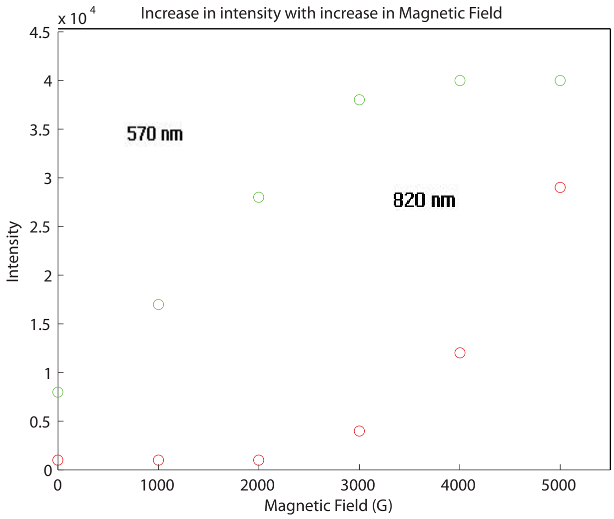


Figure 11: Increase in intensity with increasing field

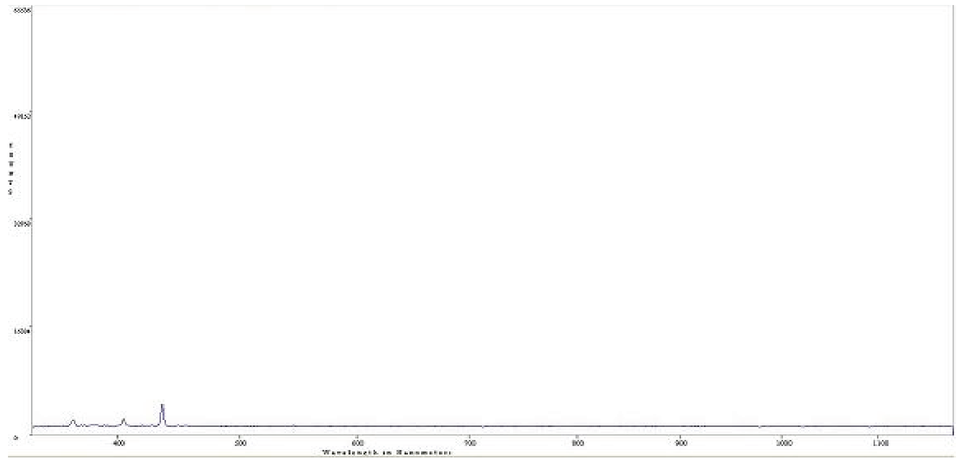


Figure 12: Spectrum of the light reaching the camera

height of electromagnet, the lower part of mercury lamp is used to observe spectrum which is not as intense as the middle part.

- About the errors and uncertainties of the experiment, the most important part is to do alignment properly. This can be done by being extremely careful in moving components and using different methods to ensure that apparatus is properly aligned. Poor alignment leads to disappointing results. Next step would be to make alignment better and using a polarizer to distinguish between the splitting under magnetic field.

- Considering the practical implications of Zeeman effect, it is used by astronomers to measure magnetic field of the sun and other stars because the distance between the Zeeman sub-level is directly propotional to the magnetic field. It is comfortably used for spectral analysis and finding magnetic field strength where it cannot be measured directly [6].

References

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