MICHELSON INTERFEROMETRY

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ABSTRACT:

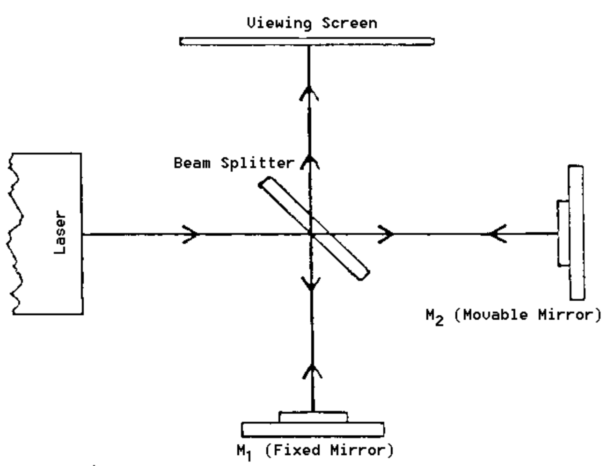
In this experiment, we will familiarize ourselves with Michelson interferometer and use it to measure some useful quantities.

INTRODUCTION:

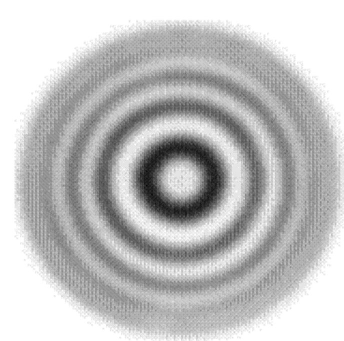
Our experiment deals with Michelson interferometer which makes us able to see interference fringes and using this configuration we will measure the wavelength of laser beam used in the experiment and also refractive index of a given glass slide.

THEORETICAL BACKGROUND:

The Michelson interferometer is the most common configuration for optical interferometry. The purpose of this configuration is to observe interference fringes by splitting a beam of monochromatic light so that one beam strikes a fixed mirror and the other a movable mirror. When reflected beams are brought back together, an interference pattern is observed on the screen. [1]



a. Michelson Interferometer



b. Interference pattern

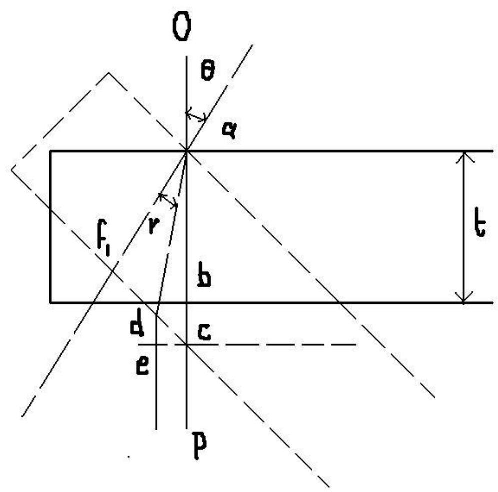
If we move the movable mirror we see the fringes entering or exiting the center of the fringe pattern, depending upon the direction in which the mirror moves. The optical path length difference between the two rays is and the maxima occurs when,

$$2d\cos\theta = N\lambda \qquad \ldots\ldots \quad (1)$$

Where N is the no. of fringes passing and λ is the wavelength of laser light used. Given a constant λ, d and N, cosθ is constant. So, if *N* is number of fringes passed by and Δd is the distance moved by M2 laser wavelength λ can be found by the following relation:

$$\lambda = \frac{2\Delta d}{N} \qquad \ldots\ldots \quad (2)$$

To find the refractive index of a glass plate, we consider the plate with refractive index n to be inserted normal to the path of one of the beams of light traversing the arms of the Michelson interferometer. This introduces an additional optical path length $(2(n-1)t$ where t is the thickness of glass plate. If this plate is rotated through a small angle θ, and number of fringes *N* corresponding to this change is counted, then we can get a fair estimation of refractive index of the glass plate. [2]



Let *OP* be the original direction of light normal to the plate of thickness t. The total optical path between *a* and *c* is *nt+bc*. After the plate is rotated through an angle θ, optical path increases to *adn + de*.

Total path would then be,

$$2(adn + de - nt - bc) = N\lambda \qquad \ldots\ldots \quad (3)$$

But

$$ad = \frac{t}{\cos r} \qquad \ldots\ldots \quad (4)$$

$$de = dc\sin\theta = (fc - fd)\sin\theta = t\tan\theta\sin\theta - t\tan r\sin r \qquad \ldots\ldots \quad (5),$$

And

$$bc = \frac{t}{\cos\theta} - t \qquad \ldots\ldots \quad (6)$$

$$\frac{nt}{\cos r} + t\tan\theta\sin\theta - \tan r\sin\theta - nt - \frac{t}{\cos\theta} + t = \frac{N\lambda}{2} \qquad \ldots\ldots \quad (7)$$

Using Snell's law, $n = \frac{\sin\theta}{\sin r}$ and re-arranging it neglecting the last term as it is very small compared to others, we get the relation to find refractive index n of glass.

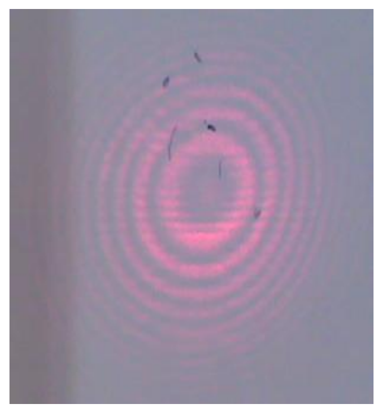
$$n = \frac{(2t - N\lambda)(1 - \cos\theta)}{2t(1 - \cos\theta) - N\lambda} \qquad \ldots\ldots \quad (8)$$

EXPERIMENTAL PROCEDURE:

The experiment can be divided into three parts.

1. Observing Interference fringes:

- HeNe laser is placed into laser mount which is bolted on the optical breadboard. Laser is aligned parallel to the breadboard.
- Beam splitter (BS) is mounted on the breadboard in front of laser.
- A fixed mirror (M1) is placed in the mirror mount assembly. The mirror is aligned such that the reflected beam from BS falls in the middle of it.
- Another mirror (M2) is mounted on the translation stage which is being operated by the computer controlled servo motor. M2 is aligned too.
- When laser is turned on, three small dots appear. Using the screws of the mirrors, these dots are overlapped and beams are aligned.
- A plano-convex lens of focal length 35 mm (L1) is placed between laser and BS and another plano-convex lens of focal length 25:4 mm on the opposite side of M1 before the target screen to enlarge and focus the image.
- Alignment is performed until concentric circular fringes are observed. Result is shown in figure.



2. Measuring Wavelength of the laser beam:

- After concentric fringes are observed on the screen, a computer software is used to move servo controlled by a fixed distance. The specifications are changed as follows:

The parameters in 'motor driving setting' are changed as follows:

In Moves-Velocity Profile:
Max. Vel. = 0.0003 mm/sec
Acc./Dev. = 0.03 mm/sec/sec

Step distance = 0.001 mm.

DC server motor has the following specifications:

Travel range = 25 mm
Lead Screw Pitch = 1 mm
Resolution = 29 nm

- Moving M2 by a distance of 0.01 mm, no. of fringes exiting the center are counted.
- Wavelength is calculated using equation (2).

RESULTS:

The results are tabulated as below:

No. of readings	Distance moved by M2 (mm)			No. of fringes displaced N
	$d_{initial}$	d_{final}	Δd	
1.	0.327	0.337	0.01	28
2.	0.347	0.357	0.01	28
3.	0.367	0.377	0.01	30
4.	0.387	0.397	0.01	30
5.	0.407	0.417	0.01	30
6.	0.427	0.437	0.01	28
7.	0.447	0.457	0.01	30
8.	0.698	0.708	0.01	30
9.	0.718	0.728	0.01	28
10.	0.738	0.748	0.01	30
11.	0.758	0.768	0.01	30
12.	0.778	0.788	0.01	30

Mean value of N = N_i = 29.285

From (2): $\lambda = \frac{2\Delta d}{N_i} = \frac{2 \times 0.01\ mm}{29.285} = 6.829 \times 10^{-6} m = 683\ nm$

$\mu^2(N_i) = \left(\frac{\sum(N-N_i)^2}{n-1}\right)^{\frac{1}{2}}$ = 0.3098 (type A) and $\mu^2(N_z)$= 1 (type B)

Combined uncertainty= $\mu^2(N) = \mu^2(N_i) + \mu^2(N_z)$ = 1.09597

And $\mu^2(d) = \frac{R}{\sqrt{12}}$ = 8.37 nm where R= 29nm

So, uncertainty in λ can be calculated using the formula, [3]

$$\mu^2(\lambda) = \left(\frac{\partial\lambda}{\partial N}\right)^2 \mu^2(N) + \left(\frac{\partial\lambda}{\partial d}\right)^2 \mu^2(d) \qquad \ldots.. \quad (9)$$

So, the final value of λ= $\lambda = (683 \pm 25)nm$

While the actual value of wavelength of He-Ne laser light is 633nm which is comparable to experimentally calculated value.

3. Measuring Refractive Index of glass:

- Thickness of the given glass slide is measured.
  t_1= 0.83 mm, t_2=0.85mm and t_3=0.83mm
  Mean t = 0.8367 mm
  t = (0.8367 ± 0.01)mm
- Glass slide is placed on the rotational stage between M1 and BS, parallel to M1 as shown in the figure.
- Glass slide is rotated by a certain number of degrees and number of fringes exiting or entering are count
- Repeated the experiment 6 times, recorded the data and calculated refractive index using equation (8).

RESULTS:

Sr. no.	Change in angle (Δθ)	No. of fringes exiting through a fixed point (N)					
		Reading 1	Reading 2	Reading 3	Reading 4	Reading 5	Mean N
1.	5	4	3	4	3	4	3.6
2.	5	12	11	12	10	12	11.4
3.	5	19	20	19	19	21	19.6
4.	5	29	26	28	28	30	28.2
5.	5	39	38	38	36	36	37.4
6.	5	50	43	43	47	46	46

The table for combined values would be as follows:

Sr. no.	Total rotation		Total no. of fringes displaced (N)	Refractive Index (n)
	θ^o(degrees)	θ^c(radians) $= \theta^o \times \frac{\pi}{180}$		
1	5	$\frac{\pi}{36}$	3.6	1.6263
2	10	$\frac{\pi}{18}$	15	1.6643
3	15	$\frac{\pi}{12}$	34.6	1.6833
4	20	$\frac{\pi}{9}$	62.8	1.6942
5	25	$\frac{5\pi}{36}$	100.2	1.6999
6	30	$\frac{\pi}{6}$	146.2	1.6950

Mean n = 1.67

While the refractive index for glass is 1.6 which is comparable to this value.

And uncertainty in N and θ is calculated as below:

$\mu(\theta)= \pm 1°$ (only type B)

$\mu(N_i) = \left(\frac{\sum (N-N_i)^2}{n-1}\right)^{\frac{1}{2}}$ (type A) and $\mu(N_z)= \pm 1$ (type B)

Combined uncertainty for every reading would then be:

$\mu^2(N) = \mu^2(N_i) + \mu^2(N_z)$

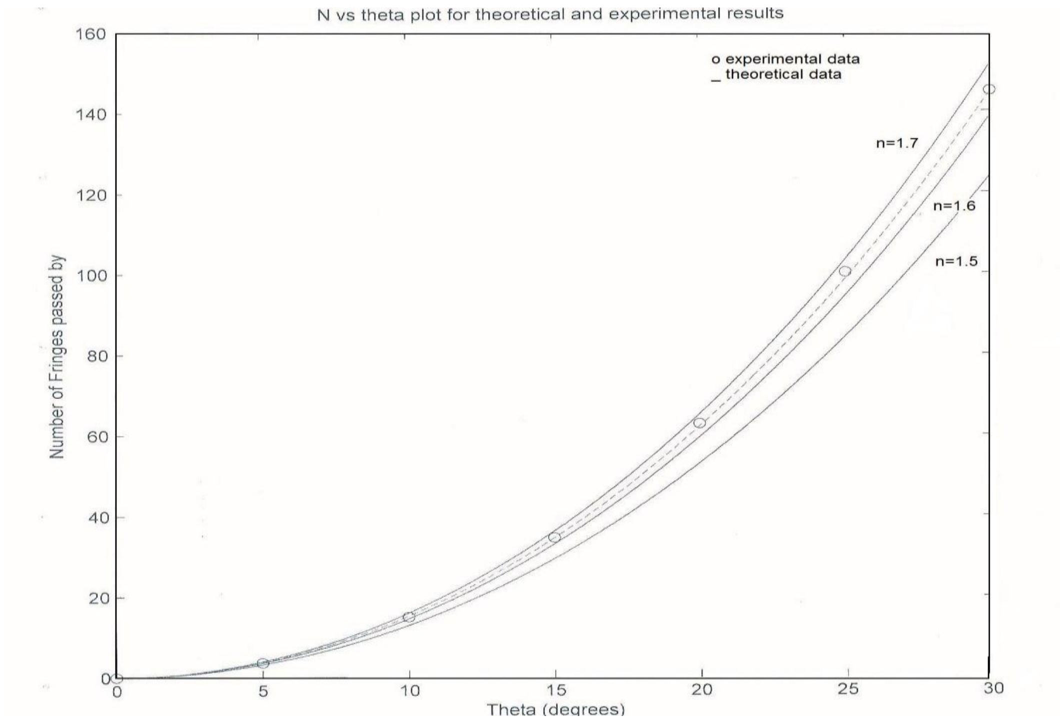
And uncertainty in n would be:

$$\mu^2(n) = \left(\frac{\partial n}{\partial N}\right)^2 \mu^2(N) + \left(\frac{\partial n}{\partial \theta}\right)^2 \mu^2(\theta)$$

Calculating it by MATLAB and showing it on a table:

Sr. no.	Total rotation θ^o(degrees)	Mean N	Type A uncertainty in N $\mu(N_i)$	Total no. of fringes displaced (N)	Refractive Index (n)	Combined uncertainty in n
1	5	3.6	0.48	3.6	1.6263	± 0.0546
2	10	11.4	0.72	15	1.6643	± 0.0521
3	15	19.6	0.72	34.6	1.6833	± 0.0401
4	20	28.2	0.68	62.8	1.6942	± 0.0289
5	25	37.4	0.56	100.2	1.6999	± 0.0261
6	30	46	0.62	146.2	1.6950	± 0.0221

The graph comparing experimental and theoretical data is shown below:



CONCLUSION AND DISCUSSIONS:

We used Michelson Interferometer to observe the interference pattern, calculated wavelength of laser beam used and measured the refractive index of a glass slide putting it in the configuration of Michelson Interferometer. The results came out to be quite comparable to the original values. The errors and uncertainties in the values are mainly caused by the sensitivity of the fringes because they are displaced even by slight movements and noise around. Rotating the glass by a small angle and calculating the exact number of fringes passed induced human error in the experiment which can be minimized by recording large number of data.

Michelson's Interferometer has many applications besides showing the interference of light. This configuration has been used for the detection of gravitational waves as a tunable narrow band filter and as the core of Fourier transforms spectroscopy. It is also used to measure trace gases in the atmosphere by Passive Atmosphere Sounding. The best known application of the Michelson Interferometer is the Michelson-Morley experiment that provided evidence for special relativity. [4]

REFERENCES:

[1] "The Michelson Interferometer", University of Central Oklahoma, 2003

[2] "Light: Principles and Measurements", Monk, pg 376-380, McGraw-Hill, 1937.

[3] Weise, K., and Wöger, W. A Bayesian theory of measurement uncertainty. Meas. Sci. Technol. 3 (1992), 1-11, 4.8.

[4] Jannsen, Michel & Stachel, John (2008). "The Optics and Electrodynamics of Moving Bodies" (PDF)