Chaos and R-L diode Circuit

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1 Abstract

In this experiment, we will use an R-L diode circuit to observe the chaotic behavior. We will analyze chaos by phase plots, time series plots, frequency spectrum and poincare maps.

2 Introduction

The spirit of nature is indeed non.linear. Most of the systems around us are non-linear that is they keep on changing with time. Some examples of non-linear relationships would include change in ocean waves due to weather change, climate change or global economy etc. In this experiment, we will study dynamics of a non.linear systems. We will see how simple experiments exhibit complex behavior and how small changes in input cause drastic change in dynamical systems. We will analyze different kind of unpredictability known as chaos. It is defined as a specific kind of deterministic behavior that is very sensitive to initial conditions. [1] Chaotic behavior shows up in systems that are essentially free from noise and possess only a few degrees of freedom. So, it actually depends on the physical aspects and the spatiotemporal properties of a nonlinear system. We will try to determine the order behind the chaotic randomness.

3 Theoretical Background

Chaotic behavior shows up in relatively simple processes. It starts with period-doubling bifurcation which is switching of the system to a new behavior with twice the period of original system at a particular value of a certain parameter. As the value of that parameter is further increased, successive bifurcations occur and the behavior of system takes a time period that is four times, then eight times and so on, finally ending in chaotic behavior. [2]

While determining chaos, we find Fiegenbaum constant. It is the limiting ratio of each bifurcation interval to the next. Every chaotic system bifurcates at the same rate. Feigenbaum constant is used to predict when chaos will arise in systems before it occurs. For every dynamic system Fiegenbaum constant is given by

$$\delta_n = \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} \tag{1}$$

$$\tag{2}$$

where λ_n is the parameter value at which nth bifurcation occurs.
When n approaches infinity, the value of this constant is,

$$\delta = 4.6692... \tag{3}$$

$$\tag{4}$$

Chaos is also determined by time series, frequency spectrum, phase plots and poincare maps. Since, chaos implies aperiodicity it can be easily investigated by the shape of its time series and so the fourier spectrum of a chaotic system is a continuum of frequencies in the Fourier domain. In phase portrait of the chaotic system there will be many distinct loops and in poincare maps, many irregularly distributed points in the map.
When we take bifurcation into account, differential equation describing a system changes. Usually a factor is subtracted from the driving force say potential in the differential equation representing the system and it no more remains linear. [3]

We will analyze a simple RL-diode circuit in detail. Circuit diagram is shown below in figure (1):

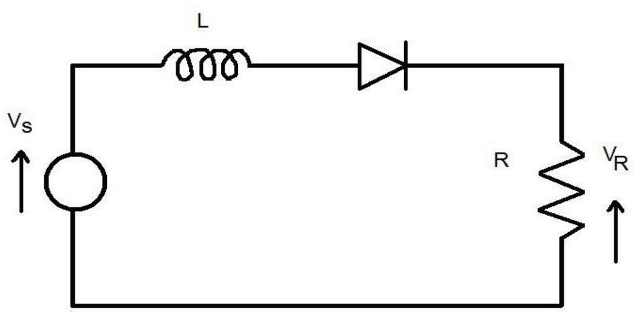


Figure 1: Circuit diagram

When diode is forward biased as shown in figure (2a), first order differential equation for the circuit is

$$L\,\frac{dI}{dT} + R\,I = V_0 \sin\omega\,t + V_f \tag{5}$$
$$\tag{6}$$

where V_0 is the amplitude of AC input voltage and V_f is diode forward voltage drop.
The solution of this equation is:

$$I = (\frac{V_0}{\sqrt{R^2 + L^2\omega^2}}) \cos(\omega\,t - \theta) + \frac{V_f}{R} + A\exp(-\frac{Rt}{L}) \tag{7}$$
$$\tag{8}$$

where $\theta = \arctan(\frac{\omega L}{R})$

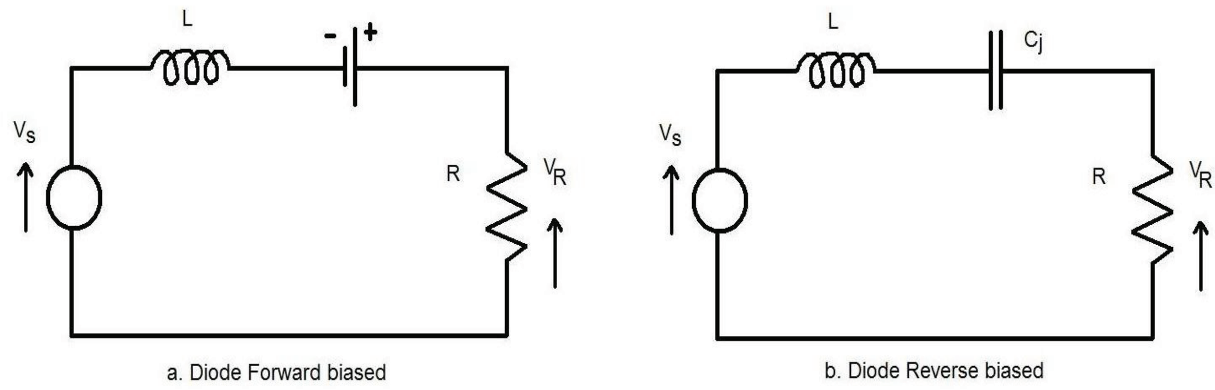


Figure 2: Conducting and non-conducting cycle of diode

When the diode is in non-conducting cycle, it behaves as a capacitor having capacitance equal to junction capacitance C_j and the circuit becomes equivalent to figure (2b).
Now, it's an RLC series circuit the circuit equation would be:

$$V_0 \sin\omega\,t = L\frac{dI}{dt} + \int \frac{I}{C}dt + IR \tag{9}$$
$$\tag{10}$$

Differentiating both sides:

$$V_0\omega \cos\omega\,t = L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} \tag{11}$$
$$\tag{12}$$

Re-arranging:

$$\frac{\omega}{L} V_0 \cos \omega\, t = \frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{LC} \tag{13}$$
$$\tag{14}$$

The solution of such an equation is a sum of transient and particular solution. Transient solution is the solution of the equation with no forced oscillation.
Finding homogeneous solution of equation (9)

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{LC} = 0 \tag{15}$$

the homogeneous solution will be of the form:

$$I_h = c_1 \exp(r_1, t) + c_2 \exp(r_2, t) \tag{16}$$

where $r_{1,2} = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L}$
and c_1 and c_2 can be calculated using initial conditions.

Now, finding particular solution of equation (9)
Assume I_p is the particular solution such that:

$$I_p = A sin(\omega\, t - \phi) \tag{17}$$

and differentiating both sides: $\frac{dI_p}{dt} = A\omega \cos(\omega\, t - \phi)$
differentiating again,
$\frac{d^2 I_p}{dt^2} = -A\omega^2 sin(\omega\, t - \phi)$

putting these values back in equation (9), we get:
$-L\omega^2 A \sin(\omega\, t - \phi) + R\omega A \cos(\omega\, t - \phi) + \frac{1}{C} A \sin(\omega\, t - \phi) = V_0 \omega \sin \omega\, t = V_0 \omega \sin(\omega\, t - \phi + \phi)$
and hence,
$(\frac{1}{C} - L\omega^2) A \sin(\omega\, t - \phi) + R\omega A \cos(\omega\, t - \phi) = \omega V_0 \sin \phi \cos(\omega\, t - \phi) + \omega V_0 \cos \phi \sin(\omega\, t - \phi)$
Now, comparing the co.efficients of $\sin(\omega\, t - \phi)$ and $\cos(\omega\, t - \phi)$ on both sides of the equation:

$$(\frac{1}{C} - L\omega^2) A = \omega V_0 \cos \phi \tag{18}$$

and

$$R\omega A = \omega V_0 \sin\phi \tag{19}$$

Now, solving for A and ϕ,
Dividing equation (19) by equation (18):
$\tan\phi = \frac{R\omega}{\frac{1}{C} - L\omega^2}$
or

$$\phi = \arctan\frac{R\omega}{\frac{1}{C} - L\omega^2} \tag{20}$$

now, squaring and adding equation (18) and equation (19), we get

$$[(\frac{1}{C} - L\omega^2)^2 + R^2\omega^2]A^2 = \omega^2 V_0^2 \Rightarrow A = \frac{\omega V_0}{\sqrt{(\frac{1}{C} - L\omega^2)^2 + R^2\omega^2}} \tag{21}$$

So, the total solution of the equation would be sum of I_h and I_p:

$$I(t) = c_1 \exp(r_1, t) + c_2 \exp(r_2, t) + Asin(\omega t - \phi) \tag{22}$$

where r_1, r_2, A and ϕ are found in equation (16), equation (21) and equation (22) respectively and constants are found using initial conditions..

In the context of such RL-diode circuit, I and $\dot{I}$ are canonical co-ordinates.
Now, we will come to the concept of diode recovery time. It is the time diode takes to stop the flow of forward current through itself as it moves to non-conducting cycle.It depends on the amount of maximum forward current that has just flown through the diode. The greater the peak forward current, the longer the diode recovery time. The formula given for diode recovery time. [4]

$$\tau_r = \tau_m[1 - \exp(-\frac{|I_m|}{I_c})] \tag{23}$$

where I_m is the magnitude of the most recent maximum forward current, and τ_m and I_c are the parameters of a specific diode.

Now, when RL diode circuit is operated at the resonant frequency, a certain amount of reverse current flows through the diode in every reverse biased cycle due to the finite recovery time. This happens because of the junction capacitance of the diode. Even when current through the diode goes to zero, some electron stays in the junction and diode conducts for almost $\tau_r = \tau_m$ which is maximum recovery time. Thus diode, instead of switching off instantaneously, switches off with a certain delay and it keeps the forward peak current smaller than the previous forward biased cycle. This causes period-double bifurcation and with increase in voltage it keeps on increasing till chaos arrives.

4 Experimental procedure

Following steps were followed during this experiment.

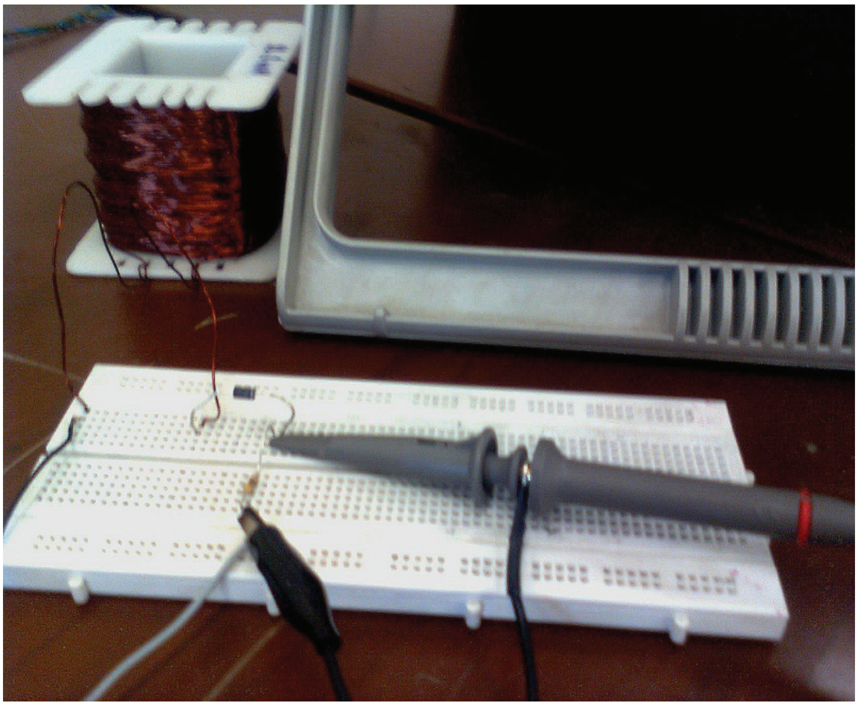


Figure 3: Experimental Arrangement

1. Circuit is made on the breadboard as in the circuit diagram in figure (1). Connections are done properly. Experimental arrangement is shown in figure (3).

2. A sinusoidal AC signal of minimum possible amplitude and low frequency is fed into the circuit and output is observed on the oscilloscope.

3. Resonant frequency of our circuit is found. It is the frequency at which the output wave amplitude on the oscilloscope is maximum. Using this frequency, junction capacitance of the diode was found.

4. Keeping frequency constant, amplitude is increased gradually and values are noted when first bifurcation, higher bifurcations and then chaos occurs. This step is repeated many times to get mean values and Fiegenbaum constant is determined. Data is recorded operating the oscilloscope in X-Y mode and photographs of the bifurcations and chaos are taken.

5. file RLD-DAQ is run on the computer which saves the output of the circuit. Data generated by LabVIEW is copied into MATLAB and phase portrait, frequency spectrum and poincare maps are plotted.

5 Results and Analysis

The resonant frequency of our circuit is found to be $90kHz$.
The junction capacitance of the diode is calculated as follows:
Since, $\omega = \frac{1}{\sqrt{LC_j}}$
where $L = 18.3 \times 10^{-3}H$
So junction capacitance would be,
$C_j = \frac{1}{L\omega^2}$
And putting the values:
$C_j = \frac{1}{(18.3\times10^{-3})H(90\times10^3)^2Hz} = 6.75 \times 10^{-9}F = 6.75nF$

As amplitude was gradually increased results shown in table 1 were obtained.

Amplitude in volts	First bifurcation	Second bifurcation	Third bifurcation	Chaos
1	1.73	5.45	6.31	7.02
2	1.73	5.45	6.45	7.03
3	1.73	5.45	6.45	6.38
4	1.74	5.47	6.30	6.96
Mean	1.73	5.45	6.38	6.86

Table 1: Successive bifurcations and chaos

Calculating Fiegenbaum constant:
Since,
$\delta_n = \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n}$
So, putting the values:
$\delta_2 = \frac{5.45-1.73}{6.31-5.45} = 4.32$

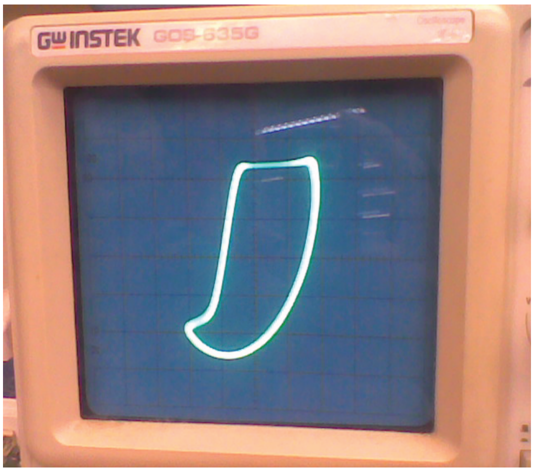
Uncertainty in V = $0.01V$

So, $\delta = (4.37 \pm 0.02)V$
which is comparable to the actual value of Fiegenbaum constant as in equation (2).

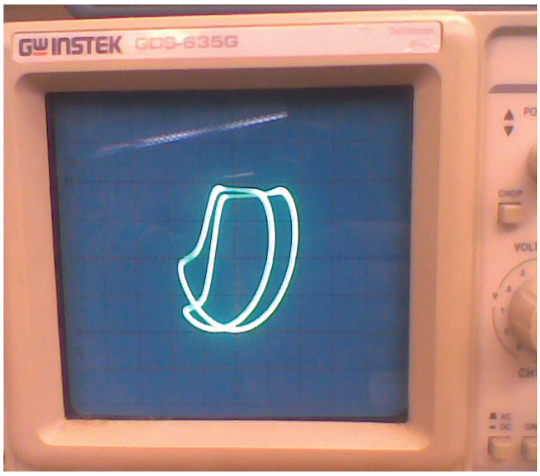
By taking input voltage on one channel of oscilloscope and voltage across the inductor on the other channel and operating oscilloscope in X-Y mode, we get the output on oscilloscope for successive bifurcations and chaos as shown in figure (4).

We can see clearly periodic behavior on no bifurcation and how aperiodicity enters into the system leading us to chaos where fuzzy bands appears after numerous bifurcations.

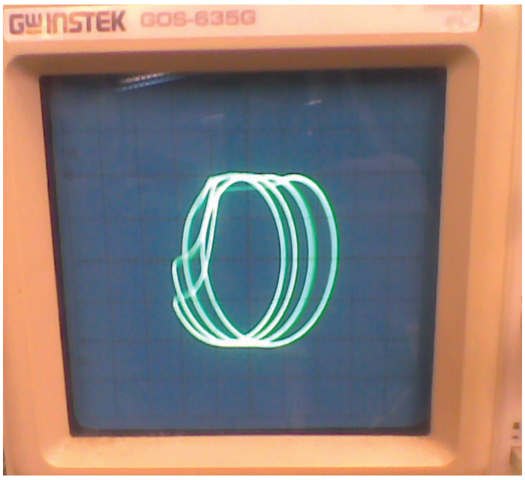
Further data was recorded into Matlab and time series plots were made shown in figure (5).



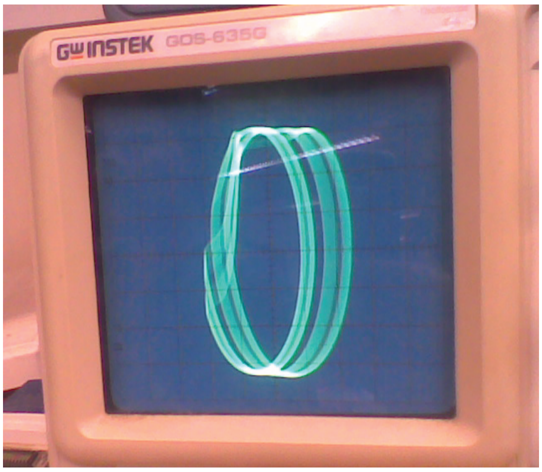
No bifurcation
Amplitude: 1.73 V



1st bifurcation
Amplitude: 5.45 V



2nd bifurcation
Amplitude: 6.38 V



Chaos
Amplitude: 6.86 V

Figure 4: X-Y output of oscilloscope

As we can see in the shape of time series plots, the difference is apparent in the periodic to aperiodic behavior of the system as it goes from no bifurcations to successive bifurcations and then chaos.

Phase portraits as plotted by Matlab are shown in figure (6):

Phase portraits are better tool to observe chaos compared to time series. We can see the difference in the number of loops as the system undergoes bifurcations. Periodic behavior corresponds to one closed loop in phase portrait. In chaotic regime, there are many distinct loops showing that the system does not approach a stable trajectory.

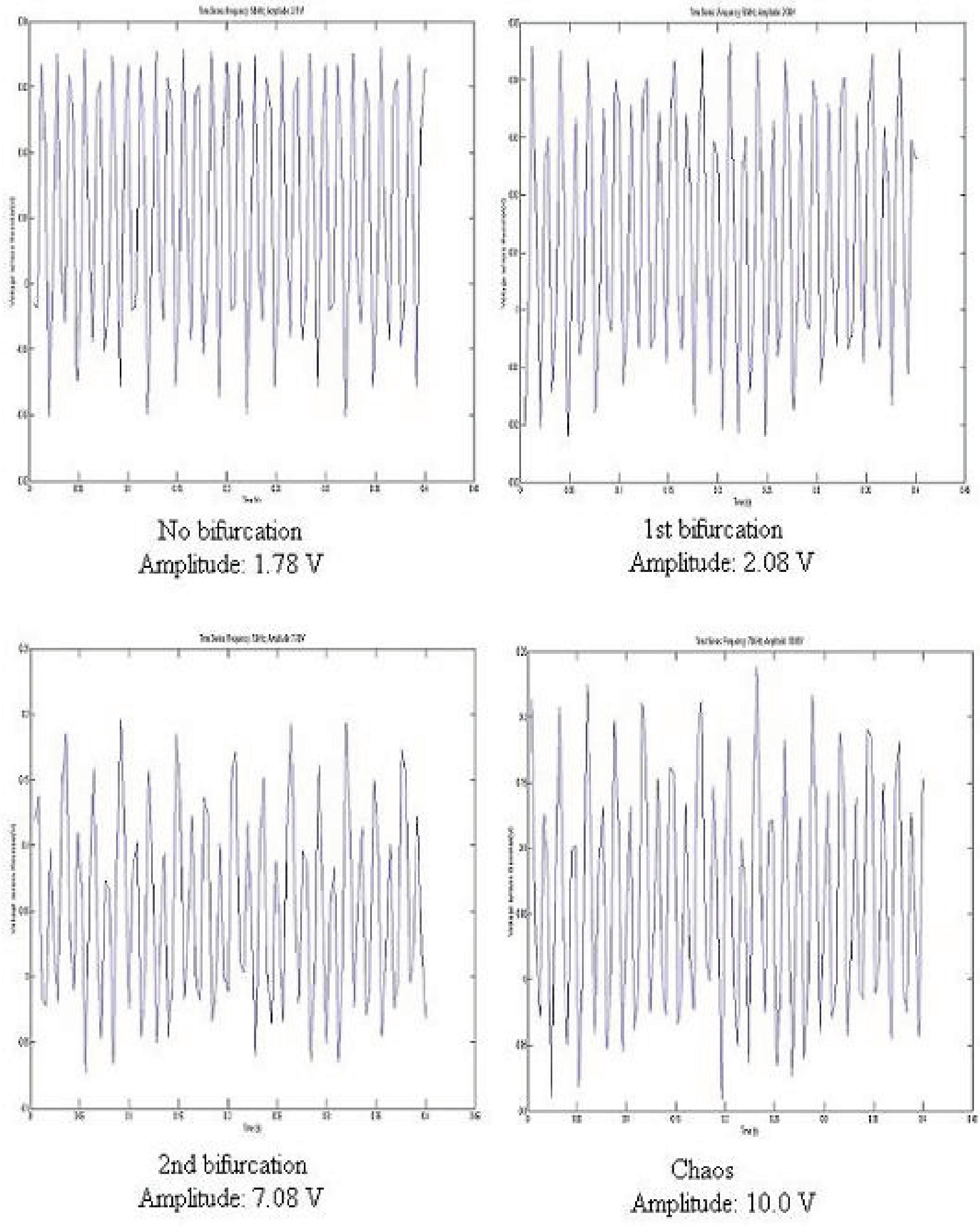


Figure 5: Time series plots

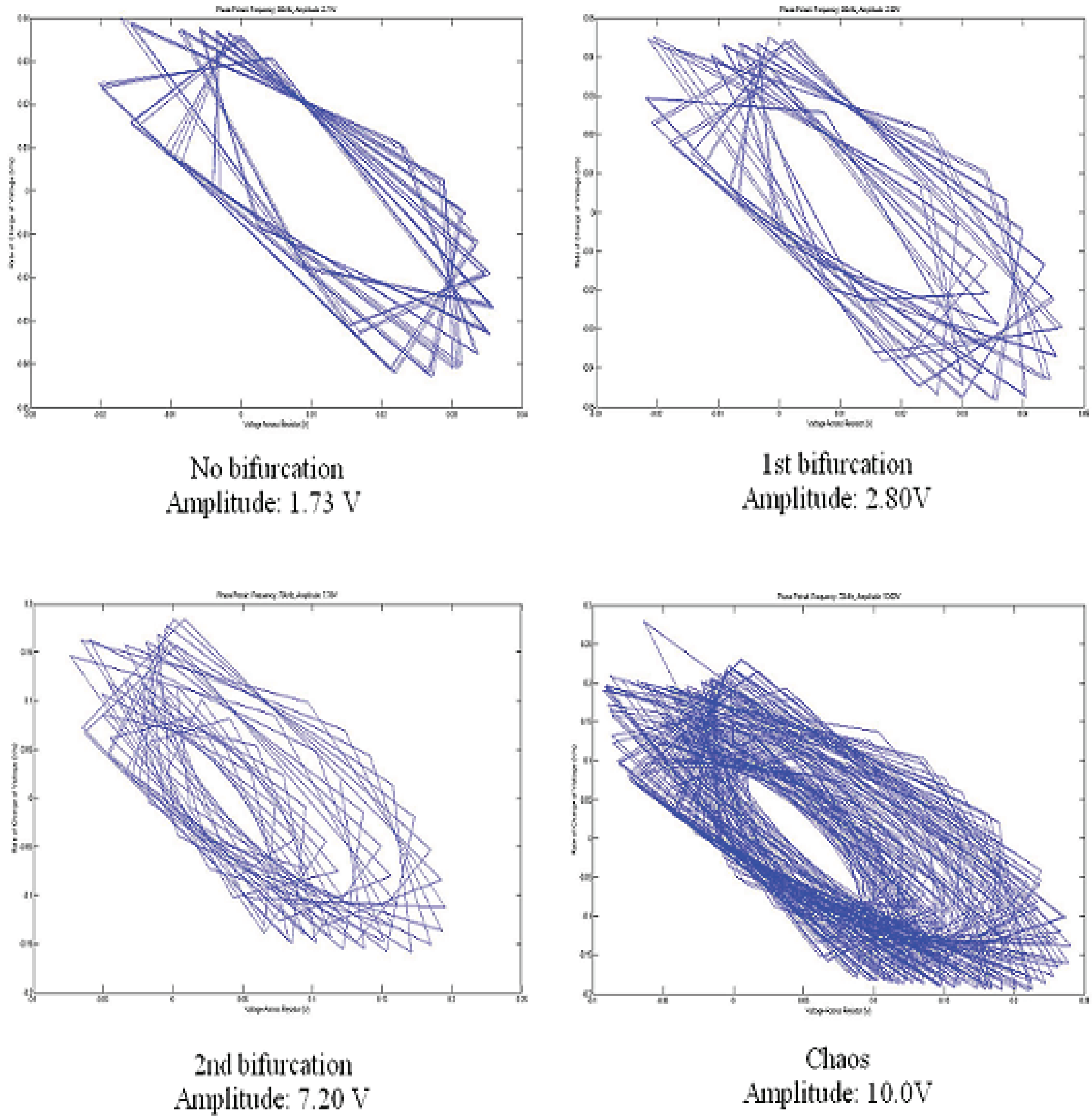


Figure 6: Phase Plots

We made fourier spectrum of chaos which appears as in figure (7):
As we can see fourier transform of the system in chaos is a continuum of frequencies.

Finally poincare plot for chaos made by Matlab is shown in figure (8):
Poincare maps sample the phase portrait of the system stroboscopically. As expected, there are many irregularly distributed points in the poincare map for system in chaos.

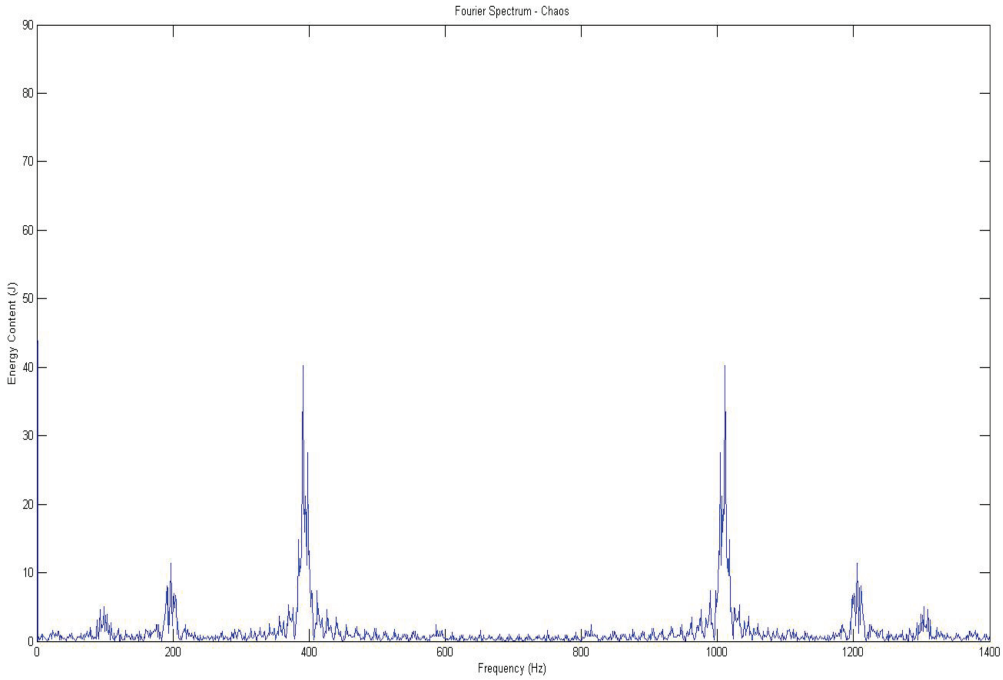


Figure 7: frequency spectrum for chaos

6 Further Discussion

- When we decrease the amplitude and notice the values when bifurcations and chaos occurs, we do see a difference. This is caused by the hysteresis in the system. In reality when system undergoes bifurcation, it jumps from one attractor to another and when we reverse this process, the threshold of this jump changes which produces hysteresis.

- We added non-linearity in our system by replacing capacitor of an RLC circuit by a diode which has a junction capacitance and finite recovery time. Greater the magnitude of the current, greater would be the non-linearity as well as chaos in the system. If we plot a graph between changing current and voltage, that would be a non-linear curve.

- While discussing the reasons for chaotic behavior specifically in RL-diode circuit, we see that it is caused by p-n junction capacitance and diode recovery time consequently. Another justification for this chaos might be fluctuations in sinusoidal voltage fed into the circuit. Since, it has a sine component, it expands into many components of different frequencies which leads to varying output behavior of the system leading to chaos.

- In complex systems, there is a lot of disordered energy and in statistical mechanics and thermodynamics, entropy is the word for disordered energy.

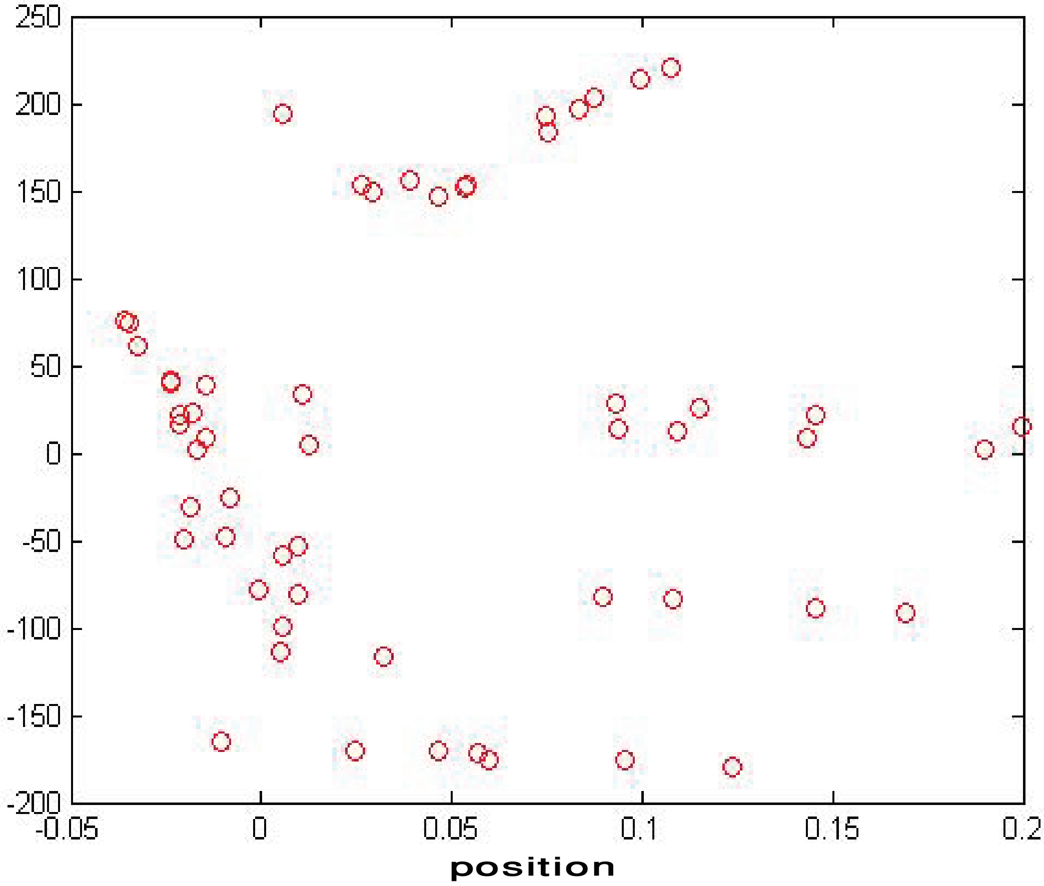


Figure 8: poincare plot for chaos

The entropy is always increasing as long as the system evolves. If the system eventually reaches equilibrium and stops evolving, its entropy becomes constant. In an information system, entropy is the natural tendency to move from order to disorder while chaos is a state of organized disorder. A system can bifurcate and attain different states from tiny differences in values. The ultimate result is chaos. So, in fact entropy predicts chaos though entropy increases smoothly while chaos is more sudden. [5]

- Chaos theory focuses on non-linear functions where the output is very sensitive to initial conditions. If we can control the initial conditions, we can still predict the output. We can use simple non-linear systems to do logic operations. Chaotic behavior has been observed in the laboratory in a variety of systems including electrical circuits, lasers, oscillating chemical reactions, fluid dynamics, and mechanical and magneto-mechanical devices, as well as computer models of chaotic processes. Chaos theory has been applied in just about every field from aerospace engineering to zoology. [6]

References

[1] http://plato.if.usp.br/fmt0308d/baranger2004.pdf

[2] http://physlab.lums.edu.pk/images/1/1d.pdf

[3] http://prl.aps.org/abstract/PRL/v49/i18/p1295$_1$

[4] http://physlab.lums.edu.pk/images/3/3

[5] http://en.wikipedia.org/wiki/Chaostheory

[6] http://ieeexplore.ieee.org/xpls/absall.jsparnumber493497